1.01 Operate with real numbers to solve a variety of problems.

a) Apply the laws of exponents to perform operations on expressions with integral exponents.

b) Evaluate absolute value expressions.

c) Evaluate radical expressions.

d) Evaluate algebraic expressions.

A. Play Krypto (B-20) with the students. A deck of Krypto cards is described as follows: three of each number 1-10; two of each number 11-17; and one of each number 18-25. Randomly select five cards. All five of these numbers will be combined, using any operations (including exponents and roots), to make an expression equal to your target number, a sixth card chosen randomly. With multiple sets of cards, groups of students can play.

Example: Suppose 23 was the target and 7, 8, 4, 3, and 16 were the numbers to work with.

\[
23 = (16 + 4 + 3) \cdot (8 - 7)
\]

or

\[
23 = 8 \cdot 3 - (\sqrt{16} \div 4)^7
\]
B. Develop or have the students develop flowcharts to simplify expressions.

Example: Choose \( x \).

\[
\begin{align*}
x & \quad \text{add 6} \\
\text{add 6} & \quad \text{multiply by 5} \\
\text{multiply by 5} & \quad \text{add 2} \\
\text{add 2} & \quad \text{stop}
\end{align*}
\]

Write the expression and solution.

C. Roll seven dice (five WHITE, one RED, and one GREEN). The outcomes of the WHITE dice must combine correctly, using any operations, to equal \( 10 \cdot \text{GREEN} + \text{RED} \).

Example: \( \text{GREEN} = 4, \text{RED} = 6, \text{WHITEs} = 6, 4, 4, 3, 1 \).

One solution is \( 46 = 6 \cdot (4 + 3) + 4 \cdot 1 \)

D. **Four in a Row**

Students will need game boards (B-24), markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another.
E. Race to the Top (B-21, 22, 23)
Enter five numbers in the triangles along the base of the triangle. To fill in space in the row above, carry out a teacher-specified operation on the numbers in the two spaces immediately below. See the example below where addition of integers is the operation.

F. Lining Up Dominoes (A-1)
Students will make a train of dominoes by successfully simplifying an expression. Blank domino sheets (B-87) can be made available so that students can create versions of the game that practice various operational and algebraic skills throughout the year, such as evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

G. Relays (B-25, 26)
Students will work in teams. Individually team members will complete a problem and share the result with the rest of the team so that a team task can be finished. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

H. I Have ... Who Has ... “ (A-3)
Students will listen, perform operations, and respond when appropriate in a round-robin format. Students will need to be able to complete operations with integers using paper and pencil. Students will use the format to create their own versions of the activity. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials that contain roots, solving a variety of equations and inequalities, and operating with polynomials.

I. Use area models to explain binomial multiplication. B-62, 63, 66, 67.
J. In early April, use the **Income Tax Rate Schedules (B-74)** to have students calculate taxes.

K. Investigate advertisements that involve percent and use the information to create problems. For instance, a car dealer offers two discounting methods. (1) Take $2000 off of the original price, then 10% more, or (2) take 10% off of the original price, then $2000 more. Which method would give the lower price to the buyer? Explain algebraically which method is best.

L. Collect five or six articles from the newspaper that use percentages. Attach each article inside a file folder and have students write problems related to their articles. Place those in the folder. In cooperative groups, have the students work the file folder problems.

M. Have students in cooperative groups make a list of as many ways as possible to use percentages. Have groups write a problem for each use. Let students report their work to the whole class. Compile the problems and use as homework, starter problems, quiz items, or test items.

N. Determine several ways to raise a number to a power using the calculator.

O. Play match game with cards matching the expansion or answer with the number raised to a power.

\[
\begin{array}{ccc}
5^2 & = & 5 \cdot 5 \\
\hline
n^3 & = & n \cdot n \cdot n
\end{array}
\]

P. Have students compare which is largest? \(100^4\), \(1000^3\), \(10000^2\)

Q. **Patterns with Exponents (B-28, 29)**
   A problem set that explores patterns generated when numbers are exponentially increased.

R. In groups, or with partners, ask the students to write a flowchart to determine the absolute value of any expression.

S. Brainstorm problem situations where absolute value is needed. Write problems using absolute value. Put the problems on a sheet and give the students the problems as a homework assignment.
T. Absolutely!! (B-27)
The object of the game is to cover as many of the integers -6 to 6 on the game board as possible and accumulate the lowest score possible. The score is the sum of the absolute values of the uncovered integers. Counters, to cover the integers on the board, and a pair of dice (different colors) are needed for each pair of players.
• Roll the dice (one is positive, one is negative).
• The player then covers the integers on the board that correspond to exactly one of the following: (1) the value on either die, not both, (2) the value of the sum, or (3) any two values having the same sum as the sum just rolled.
• A player rolls until he is unable to cover an integer. That player sums up the absolute values of the uncovered integers and records for round 1.
• The other player then rolls.
• After five rounds are played, the players total the rounds and the player with the lowest score wins.
Example: Two dice, green (positive) and red (negative). Roll 2 and -3. Cover -3. Roll 3 and -1. Cover 3. Roll 1 and -4. Cover -4. Roll 1 and -2. Cover -2. Roll 5 and -6. Cover -6. Roll 2 and -1. Cover 2. Roll 3 and -6. End of Round 1 for this player. 3, -6, -3 (3 + -6) are covered and there are no remaining pairs that will combine to make -3. This player’s score is 22, the sum of the absolute values of the uncovered integers (-5, -1, 0, 1, 4, 5, 6).

U. Use the number line (B-1) to define absolute value as the distance from zero.

V. Use base ten blocks to show the students that the square root is one “side” of a square.

\[
\begin{array}{c}
144 = 12 \times 12 \\
\end{array}
\]

Perfect squares make a square. You can approximate the square root by finding a “close” square.

\[
\begin{array}{c}
135 = 11 \times 12 + 3 \\
\end{array}
\]
W. How Do They Fit? (A-15)
Students will assemble a 3 • 3 array of puzzle pieces so that adjacent sides match mathematically. Students will be expected to create their own puzzles and have the teacher share those with the class throughout the remainder of the school year. Well constructed and edited student puzzles will provide the teacher a pool of materials to use thereafter. The format can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

X. Scientico (A-5)
Students practice translating scientific notation numbers into standard notation. Students take turns rolling three dice and constructing a number in scientific notation. Ex: 3, 6, 4 can be written 3.6 • 10^4. After recording this number on the recording chart, the student places a marker in the proper place on the game board. The student who can make three numbers in a row, column, or diagonal is the winner.

Y. Getting to the Root of the Number (A-41)
Working in pairs, students will use base-10 blocks to build incomplete squares that represent an approximate value for a specified irrational number.

Z. Bull’s Eye (B-32)
The game is played with two or more people in which each player tries to reach a specified goal number with the least number of rolls of a pair of dice. Decide who goes first. If there are more than two people playing, proceed in a clockwise manner. The teacher assigns the goal number. On each student’s turn the student will: roll the dice and compute the sum (difference, product, quotient, powers, roots) of the two numbers, either add or subtract that result from his/her cumulative total, and record the proceeds on the score sheet. The winner is the person who reaches the goal number with the fewest rolls of the dice. If no one reaches the goal number after 16 rounds, the winner is the student who is closest to the goal number. Calculators may be reserved to use only when powers and roots are the operations of choice.

AA. Use the game formats outlined on E-19, 20, 21 to practice skills and concepts with real numbers. Encourage students to create their own versions of popular (legal, of course) games.

BB. Number Crunching With Ease! (B-30)

CC. Calculator Tips (W-1, 3, 5, 9, 11)

DD. Connections (W-1, 5, 7, 9, 19, 33, 35, 41, 53, 57, 63)

EE. Warm Ups (W-2, 4, 6, 8, 10, 12, 14, 16, 22, 26, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72)

FF. Challenges (W-2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 38, 42, 44, 48, 50, 52, 58, 64)

GG. Extra Essentials (E-27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 49)

Resources for Algebra • G-6 • Public Schools of North Carolina
1.02 Operate with polynomials.
   a) Add, subtract, and multiply polynomials.
   b) Divide polynomials by monomial divisors.

A. Use algebra tiles to demonstrate addition and subtraction of like terms in polynomials.
   \((2x^2 + 3x + 2) + (x^2 - 2x + 1)\)

   \[\begin{array}{c}
   \text{\includegraphics{tile1.png}} \\
   \text{\includegraphics{tile2.png}} \\
   \text{\includegraphics{tile3.png}}
   \end{array}\]

\[3x^2 + x + 3\] is the result.

\[\begin{array}{c}
   \text{\includegraphics{tile4.png}}
   \end{array}\]

\((x^2 - 2x + 1) - (x^2 - x + 3)\)

   \[\begin{array}{c}
   \text{\includegraphics{tile5.png}}
   \end{array}\]

Take away \(x^2\), \(-x\), and add zero (+2 and -2).

\[\begin{array}{c}
   \text{\includegraphics{tile6.png}}
   \end{array}\]

Now take away 3.

\[\begin{array}{c}
   \text{\includegraphics{tile7.png}}
   \end{array}\]

\((-x - 2)\) is the result.
B. Have students write two monomials whose sum is a monomial and two monomials whose sum is a binomial. Record possible answers. Ask students what generalizations can be made. Continue by having students write two binomials whose sum is (a) a monomial, (b) a binomial, and (c) a trinomial. Again, record possible answers and ask for generalizations. Finally have students write two trinomials whose sum is (a) a monomial, (b) a binomial, and (c) a trinomial. Record answers and discuss generalizations.

C. Look at the following patterns and find the nth terms in each pattern.

I. 3^2 - 1^2 = 8
   4^2 - 2^2 = 12
   5^2 - 3^2 = 16
   ...
   nth ? ??? ??? ??? ??? ??? ???

   I. (n + 2)^2 - n^2 = 4n + 4
   II. (n + 3)^2 - n^2 = 6n + 9
   III. (n + 4)^2 - n^2 = 8n + 16

D. After trying Basketball: With the game on the line ... (A-31), try Basketball Extension 3 (B-19). This activity deals with generating polynomials that model free-throw shooting. The Basketball activities generally connect probability and statistics with algebra.

E. Model multiplying monomials with chips on the overhead. After modeling several examples, have students discuss rules they could use to multiply monomials. Ask students to discuss how this rule differs from adding two monomials.

\[2x^2y \cdot 3x^3y^2 = 2 \times x \times x \times y \quad 3 \times x \times x \times y \times y\]

\[2 \cdot 3 \cdot x^5 \cdot y^3 = 2 \times 3 \times x \times x \times x \times x \times y \times y \times y \]

\[= 6x^5y^3\]

F. Divide students into pairs. Have pairs design a puzzle to match problems with monomial • monomial and solution. Students can cut up puzzle, place in an envelope to exchange with another to solve.

G. Have students write two monomials whose product is -20x^4y^3. record possible answers on the board/overhead. Have the students make generalizations about possible answers.
H. **Multiplication With Algebra Tiles (A-35)**

Working in pairs, students will use pairs of binomials as the dimensions of a rectangle. The students will use the algebra tiles to build rectangles of given dimensions and find the area of the rectangle, the product of the binomials.

I. **Use the matrix method to multiply binomials (B-55, 57)** Make appropriate connections with multiplying using algebra tiles.

Fill in the space at the top of each column with the terms from one of the binomials. Fill in the space at the left side of the matrix for each row with the terms from the other binomial.

For \((x + 3)(x + 7)\)

```
\begin{array}{ccc}
\bullet & \times & 3 \\
x & & \\
7 & & \\
\end{array}
```

Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

```
\begin{array}{ccc}
\bullet & \times & 3 \\
\times & x^2 & 3x \\
7 & & \\
\end{array}
```
Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x²</td>
<td>3x</td>
</tr>
<tr>
<td>7</td>
<td>7x</td>
<td>21</td>
</tr>
</tbody>
</table>

In the matrix (table) of the products, add along the diagonal from right to left.

\[(x + 3)(x + 7) = x^2 + (3x + 7x) + 21 = x^2 + 10x + 21.\]

**J. Polynomial Four in a Row (B-60)**

Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for **Four in a Row** is provided (B-61) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in Algebra.

**K.** Before using algebra tiles, demonstrate binomial multiplication using an area model. See B-62, 63, 64, 65, 66, 67.
L. **Operating With Binomials (A-39)**

Students will fill in the entries for Y₁ and Y₂ with binomials and, using the calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.

M. Use the matrix (table) method to multiply polynomials. Fill in the space at the top of each column with the terms from one of the polynomials. Fill in the space at the left side of the matrix for each row with the terms from the other polynomial.

For \((2x^2 + 3x - 4)(3x + 5)\)

<table>
<thead>
<tr>
<th></th>
<th>2x²</th>
<th>3x</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Take the term from the first row and multiply it with each term at the top of the matrix and place the products in the appropriate spaces in the first row.

<table>
<thead>
<tr>
<th></th>
<th>2x²</th>
<th>3x</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x</td>
<td>6x³</td>
<td>9x²</td>
<td>-12x</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Do the same with the term from the second row and place the products in the appropriate spaces in the second row.

<table>
<thead>
<tr>
<th></th>
<th>2x²</th>
<th>3x</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x</td>
<td>6x³</td>
<td>9x²</td>
<td>-12x</td>
</tr>
<tr>
<td>5</td>
<td>10x²</td>
<td>15x</td>
<td>-20</td>
</tr>
</tbody>
</table>
In the matrix (table) of the products, add along the diagonals from right to left.

\[
\begin{array}{ccc}
\bullet & 2x^2 & 3x & -4 \\
3x & 6x^3 & 9x^2 & -12x \\
5 & 10x^2 & 15x & -20
\end{array}
\]

\[(2x^2 + 3x - 4)(3x + 5) =
\]
\[= 6x^3 + 9x^2 + 10x^2 - 12x + 15x - 20\]
\[= 6x^3 + 19x^2 + 3x - 20\]

N. The Month of Algebra (A-33)
In pairs, students will select several sets of dates in the month of Algebra and complete computations according to teacher directions. By replacing the numbers in a set with appropriate variable expressions and repeating the computations, students will be able to justify algebraically the pattern.

O. Use the puzzle formats of How Do They Fit? (A-15), Lining Up Dominoes (A-1), “I Have ... Who Has ...” (A-3), and Relays (B-25, 26) to create puzzles for student use. Whenever possible, let students create the puzzles. These formats can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

P. Use the game formats outlined on E-21, 22, 23 to practice skills and concepts with polynomials. Encourage students to create their own versions of popular (legal, of course) games.

Q. Warm Ups (W-44, 46, 50, 58)

---

**Essentials**

**for**

**Instruction**

Establish a daily routine; explain changes in the routine.

Explain the textbook format at the beginning of the course.

Provide a list of materials needed.
1.03 Factor polynomials.
   a) Find the greatest common factor of a polynomial.
   b) Factor quadratic expressions.

A. Algebra Uno
   Use sets of 100 index cards marked like the layout (B-68) of suggested cards (include four wild cards). The game is for three or four players and proceeds much like regular “Uno”. In order to lay down a card, the player’s card must show a term that has a common factor, other than one, with the card facing up on the discard pile. The player who goes out first wins. A more challenging version could include polynomials on the cards.

B. Use the matrix (table) method to factor the difference of two squares (B-56, 57).

C. Have students reverse the multiplication process to find two binomials that multiply to give these answers. (a) \( x^2 - 25 \), (b) \( a^2 - 16 \), (c) \( 4y^2 - 16 \). What pattern do they notice? Have the students write their rule for finding two binomials that have a product called the difference of two squares. Discuss how factoring is the process of reversing multiplication.

D. Demonstrate \( a^2 - b^2 = (a + b)(a - b) \). See B-69.

   Give directions orally. Cut a square and label the sides \( a \). What is the area of this square? Draw a smaller square in a corner of the first square and label its sides \( b \). What is the area of this second square?

   ![Diagram of a square with a smaller square cut off from a corner]

   Cut off the smaller square from the corner of the larger square. What is the area of this figure?
Cut the figure as shown. Rearrange these two new pieces so that they form a rectangle.

What is the area of this new rectangle? Have the students discuss the length and width of the rectangle. Ask the students to write an analysis of how this demonstrates the difference of two squares.

E. Use the matrix (table) method for factoring a quadratic trinomial (B-56, 57).

For a quadratic expression of the form \( ax^2 + bx + c \) (\( b \) can equal zero), place the \( ax^2 \) term in the upper left location (first row, first column) and the \( c \) term in the lower right (second row, second column).

What are two possible factors of \( ax^2 \)? Place the factors in the locations along the boundary for the first row and the first column.
What are two possible factors of \( c \)? Place the factors in the locations along the boundary for the second row and the second column.

Multiply the binomials that are now in the boundary. Do the two diagonal products add up to box? If not, go back and adjust the factors for \( c \) and/or \( ax^2 \).

---

F. Polynomial Four in a Row (B-60).

G. Fill in each square with a digit 0 - 9. You may use each digit only once (B-70, 71). Similar puzzles are available commercially.

\[
\begin{align*}
x^2 - x - \square &= (x + 2)(x - \square) \\
x^2 - 1\square x + 1 \square &= (x - 2)(x - \square) \\
x^2 + \square x + 1 \square &= (x + \square)(x + 2) \\
x^2 - \square x - 24 &= (x - 6)(x + \square)
\end{align*}
\]

H. Factoring Trinomials with Algebra Tiles (A-37)

Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.
I. Use the puzzle formats of How Do They Fit? (A-15), Lining Up Dominoes (A-1), “I Have ... Who Has ...” (A-3), and Relays (B-25, 26) to create puzzles for student use. Whenever possible, let students create the puzzles. These formats can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

J. Use the game formats outlined on E-21, 22, 23 to practice skills and concepts with polynomials. Encourage students to create their own versions of popular (legal, of course) games.

K. The difference of two squares can be used with mental math. To multiple 24 and 26, the product can be found using (25 - 1)(25 + 1) = 625 - 1 = 624. Have the students try 17 • 13.
   Using your overhead calculator, enter \( y = x^2 \). Set your table generator to begin at ten and increment by five (or students can do this with individual calculators). Show the table of squares to the class. Have each student write a multiplication problem that would use the difference of squares and one of the squares in the table. Take up the cards and form two teams. Read a problem and call on a person to give the product using only mental arithmetic. Each correct problem is worth one point.

L. Have the students investigate the relationship of factoring the difference of two squares and the graph of a quadratic function.

```
<table>
<thead>
<tr>
<th>Graph</th>
<th>Factors</th>
<th>Vertex</th>
<th>x-intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 - 1 )</td>
<td>((x+1)(x-1))</td>
<td>(0, -1)</td>
<td>(-1, 0), (1, 0)</td>
</tr>
<tr>
<td>( y = x^2 - 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x^2 - 9 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x^2 - 16 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

What happens to \( y = x^2 + 4 \)?

M. Have students review types of factoring by writing about the relationship of factoring to multiplication. Use an example from each type of factoring that we have studied. Explain the relationship between multiplying the factors to obtain a polynomial and factoring the polynomial into the factors.

N. Make the Message (B-72)
   Factor quadratic and rational expressions to create a message. Using the same format, create similar messages, copy, and assign for homework, review, or extra credit.

O. Warm Ups (W-56, 68)
2.01 Use formulas and algebraic expressions (from science, geometry, statistics, etc.) to solve problems.

A. Ask the students to talk to their parents and others to find out what kinds of formulas they use in life. In groups during class compile a list of the formulas used. Match the formula to who uses it. Have each group design a poster to illustrate the formula. Solve for different variables in the formula.

B. Hand out formulas on index cards. Ask the students to write a problem for the formula he/she receives. Switch cards and repeat the process.

C. Sports offer a particularly rich context for using formulas. Let students investigate formulas in sports where they are most interested and create problems based on those formulas. Here is an example.

According to the NCAA, Richie Williams, Appalachian State University, was the highest rated quarterback in North Carolina during the 2004 football season. The NCAA uses the following formula to measure the passing efficiency of quarterbacks.

\[(\text{yards per attempt}) \times 8.4 + \text{completion percentage} + \text{touchdown percentage} \times 3.3 - \text{interception percentage} \times 2\]

For Williams that would be:

\[
\frac{3109}{350} \times 8.4 + (100 \times \frac{234}{350}) + (100 \times \frac{24}{350}) \times 3.3 - (100 \times \frac{10}{350}) \times 2 = 158.4
\]

Find the appropriate statistics for the college quarterbacks in North Carolina from the most recent football season. Rank them using the formula for passing efficiency.
D. The formula to convert Celsius (°C) to Fahrenheit (°F) is \[ F = \frac{9}{5}C + 32 \]. Derive the formula to convert Fahrenheit to Celsius. Have the students graph the formula on the calculator and create a table to convert between the two temperature scales.

E. Use the game formats outlined on E-19, 20, 21 to practice skills and concepts with formulas. Encourage students to create their own versions of popular (legal, of course) games.

F. Connections (W-1, 5, 13, 29, 33, 39, 47, 53, 57, 63, 69)

G. Warm Ups (W-10, 12, 18, 20, 24, 28, 30, 32, 36, 48, 50, 54, 60, 62, 64, 66, 70)

H. Challenges (W-36, 68)

I. Extra Essentials (E-32, 34, 39, 40, 41, 42, 43, 44, 45, 47, 49)

---

**Essentials for Instruction**

- Have frequent review.
- Summarize and allow for questions.
- Circulate and assist.
- Use diagrams and other visual aids.
- Use samples of finished product as models.
- Clarify criteria and format when giving written assignments.
- Use an uncluttered, consistent format for worksheets.
2.02 Describe, extend, and express algebraically a wide variety of geometric patterns.

A. **Patterns in Perimeter (B-35, 36, 37, 38)**

Students will use perimeter to generate data, find the algebraic expression for the linear pattern, and graph the data that are generated. Although it is expected that students will work individually or in pairs on the pattern sheets, overhead versions of two of the problems are provided for whole class discussion. Using pattern blocks to build the figures is particularly helpful for many students’ understanding.

B. **Toothpick Triangles (A-27)**

Have students on graph paper graph the ordered pairs. Discuss the meaning of slope geometrically and numerically from the data.

C. Have students investigate a pattern for surface area with linking cubes. Give each group fifteen linking cubes. First, have students connect five cubes as shown in figure 1 and find the surface area. Students build the second shape using 10 cubes; record the surface area then build the third shape. Have students describe verbally how they found the surface area, then write a formula for the pattern.

\[
\text{Formula: } \text{SA} = (2N + 2) \cdot 5 + (4N - 2) = 14N + 8
\]

Extension: How would the formula for the surface area change if two cubes are used for each step? three cubes? four cubes? Is there a pattern present? How would you describe it?

<table>
<thead>
<tr>
<th>N</th>
<th>Calculate</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4\cdot5 + 2</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>6\cdot5 + 6</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>8\cdot5 + 10</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>10\cdot5 + 14</td>
<td>64</td>
</tr>
</tbody>
</table>
D. Your mom is designing a flower bed and will use bricks (gray squares) around the bed. Each white square represents a flower plant that will be planted in the bed. Find the formula for the number of bricks that will be needed for n plants.

(Answer: \(2n + 6\))

Now, you know bricks aren’t really square but rectangular. Write the formula when \(n + 3\) represents the brick! (Answer: \(n + 3\))

Make up another problem with increasing squares. Exchange papers with a partner. Write the formula for the new problem.

E. Ask the students to draw two congruent right triangles with one vertex in common as shown.

[Diagram of two congruent right triangles]

Change the vertical angles by changing the lengths of the opposite sides. Make a table of the changes and look for patterns. Use the Pythagorean Theorem, area, perimeter, and other geometric aspects of the figure to look for patterns.

F. A rectangular prism is built with 100 centimeter cubes. Determine how many different rectangular prisms can be made keeping the volume constant. Develop a table and look for patterns. Compare surface area and volume.

G. **Patterns in Area and Volume (B-39, 40, 41)**

Students will use surface area and volume to generate data, find the algebraic expression for the pattern, and graph the data that are generated. Using blocks to build the figures is particularly helpful for many students’ understanding. There are linear, exponential, and quadratic patterns among the sequences. Only after students have described the patterns in arithmetic terms should students use the curving fitting utilities on their calculators. The method of finite differences can be used to identify quadratic patterns. (With Algebra II and Technical Math 2 students, follow up finite differences by setting up the matrix equation and solve it to derive the quadratic expression.)
H. Draw a square of any dimension. Draw a smaller square in one corner of the larger square with a side $\frac{1}{3}$ the length of the larger square. Calculate the areas and perimeters of the two squares. Start over with the same original square and draw a new square in the corner $\frac{1}{4}$ the length of the larger square. Calculate the new area and perimeter. Repeat with squares $\frac{1}{3}$ and $\frac{1}{2}$ the length of the original.

Discuss the relationships between the large and smaller squares. State any generalizations algebraically.

I. **Extra Essentials** (E-29, 37, 45, 46, 47, 48)

<table>
<thead>
<tr>
<th>Essentials for Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use concise directions, written and oral.</td>
</tr>
<tr>
<td>Provide time frame for long-range assignments.</td>
</tr>
<tr>
<td>Use a multisensory presentation.</td>
</tr>
<tr>
<td>Give oral and written directions together.</td>
</tr>
<tr>
<td>Emphasize where directions are located.</td>
</tr>
<tr>
<td>List steps (oral and written) necessary to complete assignment.</td>
</tr>
</tbody>
</table>
3.01 Translate word phrases and sentences into expressions and equations and vice versa.

A. Have each student write a numeric expression (phrase or sentence) in words on an index card. Switch cards with a partner. On a separate card, write the expression in symbols and numerals so that each of the original cards has a match. Check and recheck to make sure the cards have accurate matches. Take up the cards, shuffle them, and pass the cards back out to the students. Have the students again look for matches. (You may also take up the cards in small groups.)

B. Have each student choose a number. Then: add 5, double the result, subtract 4, divide by 2, and subtract the number you start with. The result should be 3. Use counters or blocks to show how the result is always 3. Let a particular color of counter or shape of block represent an undetermined number. Show how the result is always 3. (B-13) Use algebraic notation to show why the result is always 3. Try several other tricks, first illustrating with counters or blocks and then with algebraic notation. Have students create their own tricks and share with the class. Use the student creations as starter problems, classwork, homework assignments, or even test items. Be sure that the algebraic representation and explanation is a part of the assignment.

C. After trying Basketball: With the game on the line ... (A-31), try More Basketball: Extension 2 (B-18). The Basketball activities generally connect probability and statistics with algebra.

D. Connections (W-23)

E. Warm Ups (W-6, 8, 10, 16, 18, 20, 30, 34, 60, 72)

F. Extra Essentials (E-42)
3.02 Identify properties and relationships of data in tables, graphs, and equations.

A. Turn your classroom into a coordinate plane. Each desk should have a location on the plane. Give the students a card with a “location” (ordered pair) on it. Have the students one-at-a-time find their places. You may wish to give students their cards as they walk into the classroom.

B. Create a coordinate plane in the classroom, on the blacktop, or some large area. Have the students do the “Algebra Walk” by walking to the points you ask them to find. Ask the students to work in pairs. Give each pair an index card with an ordered pair to locate on the coordinate plane. Ask one to be a walker and the other a checker.

C. Create a coordinate plane on a shower curtain liner using markers or gym tape. Place velcro dots at the intersections. Give the students points to locate. Place a dot at that point. (Note: A shower curtain liner is inexpensive and can be used repeatedly, especially if gym tape is used.)

D. The Stock Market

   Using a newspaper with comprehensive stock market coverage, have students familiarize themselves with the terms and layout, pricing, and share profit for at least ten stocks with a bullish trend and ten stocks with a bearish trend. Use North Carolina companies where possible. Have students research the Capital Asset Pricing Model and the Price/Earnings Ratio that brokers and analysts use to study companies and apply them to their selected stocks. How do these analytical models determine good investments? What other information is necessary to make good investment decisions?

E. Have students in pairs write an application problem that could be represented by a table of data and two to three questions that can be answered from a graph of the data. Here is an example:

   Marta and Gene persuade their Dad to buy them a lawn mower for a summer business. They will pay their Dad back the cost of the lawn mower which is $190 and will charge $7 an hour for mowing a yard. How many hours must they work to make a profit of $200? After how many hours will they break even?

   In the computer lab, have each pair design a spreadsheet for their problem. Have the students design a table of values, enter an equation to calculate more values and draw a line graph. Or have students write an equation for their data, look at the table and graph on a calculator.
F. Calculator Tips (W-27, 29, 45).


H. Teacher to Teacher (W-51)

I. Warm Ups (W-14)

J. Extra Essentials (E-24)

---

**Essentials for Instruction**

- Stress accuracy.
- Use “hands-on” activities.
- Provide direction when students have several options.
- Give time for students to organize.
- Teach cues and other listening skills.
- Allow time for explanations before giving assignments.
3.03 Define and distinguish between relations and functions, dependent and independent variables, domain and range.

A. Use the grid activities from 3.02A, B and ask the students to be the “points” on a variety of lines. Have them determine whether the line(s) are functions.

B. CBLs and similar electronic devices should be used whenever possible as generators of “real world” data for students’ inspection and analysis. How Hot (Cold) Is It? (A-11) is one such activity. Although the only curve-of-best-fit expected in Algebra I is a linear one, should sets or subsets of data collected be exponential or quadratic in nature, it would be appropriate for students to try the other best-fit utilities of their calculators.

C. Floordinate Plane
   - Create two sets of cards, x-coordinates and y-coordinates, each numbered from -10 to 10.
   - On the floor, use masking tape to lay out the coordinate axes.
   - Give each student a card from each set and have them stand at the location indicated by their cards.
   - Students exchange x- and y-coordinates and locate the ordered pair on the coordinate plane diagramed on the floor.
   - Identify the new set of locations as a relation.
   - Investigate to see whether the relation is a function. Have all the students face the x-axis. If none of the students have another student standing in front of them, then they are a function.
   - Repeat the process.

D. The Train Analogy
   Think of a function in terms of the following analogy. Consider the domain to be a set of people who ride the train to work each day. Additionally think of the range as the set of station stops along the train line. The “function” of the train is to deliver the passengers to their respective destinations. It is possible that two or more people will get off at the same station; however, it is not possible for one person to get off at two different stops. For many students this analogy clarifies the concept that each domain value, x (a person), is associated with one and only one value, y (a station stop).

E. The Stock Market
   Using a newspaper with comprehensive stock market coverage, have students familiarize themselves with the terms and layout, price, and share profit for at least ten stocks with a bullish trend and ten stocks with a bearish trend. Use North Carolina companies where possible. Have students research the Capital Asset Pricing Model and the Price/Earnings Ratio that brokers and analysts use to study companies and apply them to their selected stocks. How do these analytical models determine good investments? What other information is necessary to make good investment decisions?

F. Connections (W-3, 5, 9, 15, 17, 23, 27, 31, 35, 43, 45, 47, 49, 51, 61, 65, 67, 71)
3.04 Graph and interpret in the context of the problem, relations and functions on the coordinate plane. Include linear equations and inequalities, quadratics, and exponentials.

A. Highs and Lows

Students need to make a chart in their notebook to record the daily high and low temperatures for a one-month period. Since there are two quantities (time and temperature) that they will be dealing with, students need to discuss and reach an agreement on which quantity is dependent and which is independent. At the end of the month, students need to create three graphs. (daily high temperatures, daily low temperatures, and a composite of the two.) Students will find a best-fit line (manually and using the calculator) and make predictions for the next month based on their equations. Students should calculate the average high and low temperatures for the month and compare with their graphs. Students should use the calculators to explore the other best-fit possibilities available. (The actual best-fit function would be trigonometric, sine or cosine. This would be more obvious for data collected over the course of a year.)

B. When given the general form of a quadratic, \( y = ax^2 + bx + c \), students should be able to discuss changes in the graph of the equation as \( |a| \) increases or decreases, changes in the graph when \( a \) is positive or negative, when to expect a maximum or minimum value for the equation for a given value of \( a \), how \( a \) is related to the problem being addressed, changes in the graph of the equation as \( c \) increases or decreases, how \( c \) is related to the problem being addressed. See The Picture Tells the (Quadratic) Story (B-76, 77).

C. When given the general form of an exponential, \( y = ab^x \), students should be able to discuss changes in the graph of the equation as \( |a| \) increases or decreases, the shape of the graph when \( b > 1 \) or \( b < 1 \), and \( a \) and \( b \) as they relate to the problem being addressed.

D. Give students a situation and ask them to sketch a reasonable graph by just drawing the basic shape. Have students explain their graphs. Here are examples:

- The amount of money you earn on a part-time job and the number of hours you worked.
- The number of people absent from school each day of the school year.
- An individual’s height as he ages.
- The height of a baseball as it is hit in the air.
- The amount of daylight each day of the year.
- The distance a car travels at a constant rate over a three-hour time period.
- The amount of money in a savings account over an extended period of time.
E. Create a graph on the human grid described in 3.02B. Give each student a different number for \( x \). Have the students begin by walking a line that represents ordered pairs that represent \( y = x \), then everyone add 2 to \( x \). Go back to the original line and take the absolute value of \( x \). Continue using other modifications of \( x \).

F. Use the human grid to graph an equation. Ask the students to work in groups. Give each group a different equation.

G. Pair off students. Have one student describe a graph in as accurate a manner as possible and have the other student draw the graph. No peeking allowed until the drawing is finished.

H. Have students sketch a graph for the following situations:
   A mail carrier on a rural route must slow down as she approaches a mailbox, stop briefly to place the mail in the box, then continue to the next stop. Sketch a graph of the time verses distance as she delivers mail to three customers, has no mail for the next three deliveries and then leaves mail for two more persons.

I. Have students sketch graphs on the same axis to represent the following three students. Let \( x = \) time and \( y = \) distance.
   - **Martin** walks to school each morning leaving at 8:00 and arriving at school at 8:30. One morning, he walked for 8.5 minutes then returned home to pick up his homework. He continued back to school.
   - **Juanita** (Martin’s sister) drives her car to the same school.
   - **Terry** (Martin’s brother) usually leaves fifteen minutes later but must run to school and arrives about 8:35

J. Collect a variety of graphs from newspapers, magazines, and other sources. Give a graph to each student. Ask them to write a newspaper article interpreting the graph.

K. **Gulliver’s Clothes** (A-13)
   This activity will have students working in small groups collecting and recording the circumference of each group member’s thumb, wrist, and neck. The data will be plotted on a scatterplot and compared to a linear function. Students will use the scatterplot and linear function to make predictions.

L. Give students the following graphs and have them write a story for each:

---

*Resources for Algebra • G-27 • Public Schools of North Carolina*
Sara is fishing:

M. Give students graphs generated from spreadsheets (or have them generate the graphs). In pairs or groups, ask students to write 4-5 questions to be answered from their graphs. Have students exchange papers and answer the questions.

N. After the appropriate research at the grocery store, have students graph on the calculator an equation to represent the cost of a particular food item at d dollars a pound or per item. \((y = d \cdot x)\) Students can use the integer screen and trace coordinates to determine the \(x\) and \(y\) values. Discuss why this relationship is a function.

O. Create a coordinate plane and demonstrate the line \(y = x\). Give each student a card with an ordered pair written on it. In small group or with partners, ask the students to determine where their points lie and their relation to the line \(y = x\).

P. The Top Fifteen

Give each student a list of 15 top television shows, movies, CDs, etc. Ask each student to rank the list with 1 being the one most liked and 15 being the least liked. With a partner ask the students to graph their rankings as ordered pairs. \(ER\) was a television show that consistently finished in the top 15 during the 1998-99 season. Suppose Student A rates the show a 3 and Student B rates it a 5. Then \(ER\) would be graphed as the ordered pair (3,5). What would the \(y = x\) line mean in this context? What does a widely scattered set of points mean? What does a set of points close to \(y = x\) mean? Ask each pair of students to describe and interpret their results.

Q. Determine how many “perfectly square people” you have in your class. Ask the students to measure their height and arm span. Compare the ordered pairs to the line \(y = x\).

R. Have students graph and explain applications of linear equations. Here is an example. Tickets to the Carolina Theatre are $6 for adults and $3 for children. If the theatre would like to have a sales income of $600, the equation to model possible solutions is \(6x + 3y = 600\). Graph the equation. Choose three ordered pairs that are solutions and explain what each represents in terms of the number of adult and children tickets sold.
S. Have students investigate what happens if you change the scales of the axis when graphing. For instance, on graph paper draw graphs of \(10x + 5y = 15\) using different scaling methods. (a) Scale both axes by one, (b) scale both axes by five, (c) scale the \(x\)-axis by one and the \(y\)-axis by five, and (d) scale the \(x\)-axis by five and the \(y\)-axis by one. Compare your results.

T. Write sets of equations that you think will produce parallel lines. Write a set of equations that intersect the \(y\)-axis at (0,5).

U. Have students enter a table of data from a linear application on a spreadsheet and graph the data. Students can title and label the axes. Have students share printouts with the class and explain the graphs.

V. **Calculator Tips** (W-21, 23)

W. **Connections** (W-23)

X. **Warm Ups** (W-24, 26)

Y. **Extra Essentials** (E-25, 35)

---

**Essentials for Instruction**

- Provide specific feedback on completed work.
- Work from concrete to abstract.
- Allow some written assignments to be done as a group project.
- Use small groups to discuss main ideas.
3.05 Determine and use slopes of linear relationships to solve problems.

a) Find the slope of a line given the graph of the line, an equation of the line, or two points on the line.

b) Describe the slope of the line in the context of a problem situation.

A. Have students work in pairs. Each pair folds a sheet of notebook paper in half and then tears the sheet into two pieces. Then fold each piece into thirds. Each person draws a graph in the middle frame. Students switch papers. Ask them to write on the left frame the slope and y-intercept of the graph and on the right frame the equation of the line.

B. Students can discover the y-intercept formula using calculators. For instance, have students graph each set of equations. Then ask students to describe how the lines are alike and how they are different.

\[
(1) \quad y = 2x + 1, \quad y = 2x + 5, \quad y = 2x - 2 \\
(2) \quad y = 3x + 2, \quad y = 3x - 1, \quad y = 3x - 3
\]

Ask students which values in the equation appear to represent the slope and which the y-intercept. Then ask students to write an equation with slope 5 and a y-intercept of 1. Students can explain their reasoning.

C. Give students two ordered pairs to graph a number line on graph paper. Ask them to find the slope geometrically and then using the slope formula.

D. Play Guess My Slope and Y-Intercept (B-46), a game using a TI-81/82 program.

E. Boards and Bands

Give each student (or group of students) a geoboard and set of bands. Have the students mark the x- and y-axes on the geoboard with an erasable marker. Scales can vary so that each peg can represent one unit, two units, five units, and so on. Locate points on the geoboard and discuss how two points are necessary to identify a line. Selecting two points, find the slope of the line which is described by the points. Determine the equation of the same line. Use the calculator to check and compare results. Select other points and repeat the procedure.
F. **What’s in a Letter?**
Describe the segments that comprise a capital letter in terms of their slope. Example: the letter contains a horizontal segment, a segment with a positive slope, and a segment with a negative slope. What is the letter? (A) After the teacher has described several letters in this fashion, let students write and share descriptions of upper and lower case letters with each other. Enlarge and overlay letters on a grid and let students numerically discuss the segments’ slopes.

G. **Exploring Perpendiculars**
Draw several segments on graph paper and give each student a copy. (B-47) Have the students find the slope of each segment and record. Have the students fold the graph paper so that the fold is perpendicular to the original line. (This can be done by matching the endpoints and folding.) Have the students find the slopes of the folds and compare the results with the slopes of the corresponding segments.

**Used Cars**
H. Have students ask 10-12 teachers the ages and mileages for their cars. Have the class draw a scatter plot and a line of best-fit. Ask students to estimate a slope and interpret the slope in the context of the situation.

I. Give the equation \( y = 3.5x + 25 \), have students create a situation modeled by the equation. For instance, let the equation describe the relationship between the number of martial arts classes (\( x \)) and their cost (\( y \)) at the recreation center. What does the y-intercept represent? What does the slope represent?

J. **Connecting Units of Measure** (A-25)
Students will measure several objects in the classroom using both centimeters and inches. Students will plot corresponding pairs of measurements on a graph and interpret the information to determine the relationship between centimeters and inches.

K. **Making Sense of Slope**
The handouts B-48, 49, 50 contain five problems that are related to graphing and slopes of lines. Each problem contains a graph or chart that the students are to analyze and answer questions about. Work problem #1 as a class to give students a good idea of what is expected. It would be appropriate for students to work in groups of three or four to complete the assignment.

L. **Drive Time** (B-73)
Tina’s dad wishes to calculate the number of hours it will take to drive to certain vacation spots. He estimates that he can drive at an average of 58 mph.
M. Give students a table of data. Ask them to calculate the slope and interpret. Example:

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Wages Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>28.75</td>
</tr>
<tr>
<td>10</td>
<td>57.50</td>
</tr>
<tr>
<td>15</td>
<td>86.25</td>
</tr>
<tr>
<td>20</td>
<td>115.00</td>
</tr>
</tbody>
</table>

N. Connections (W-47, 53)

O. Challenges (W-70)

P. Extra Essentials (E-35)

---

**Essentials for Instruction**

- Orally emphasize key words.
- Provide clear copy (worksheets, handouts, etc.).
- Teach abbreviations germane to the course.
3.06 Write the equation of and graph linear relationships given relevant information:

a) Slope and y-intercept
b) Slope and one point on the line
c) Two points on the line

A. Discuss how to graph a line using the slope and y-intercept through an application. Here is an example. A health club charges a $75 membership fee and $50 annual dues. Create an equation in slope-intercept form where \(x\) is the number of years and \(y\) is the total cost. Graph.

B. Give students an equation in standard form to graph on the calculator. Show only the final graph on the overhead. Students can check to see whether they have the same graph.

C. Sports are appropriate contexts to create problems involving algebra. Here is an example. During the 1999 baseball season, each team in Major League Baseball played 162 games. On April 30, the Boston Red Sox had played 22 games and won 11. By May 31, Boston had played 50 games and won 31. Create a linear equation in slope-intercept form that describes this trend. Define the slope of your equation with respect to the quantities being discussed. Explain how you created your equation and determine the number of games Boston should win according to your model. How many games did Boston actually win that season? How do your “expected wins” compare with the actual wins? Explain the difference, if any.

D. Toothpick Triangles (A-27)
   Have students on graph paper graph the ordered pairs. Discuss the meaning of slope geometrically and numerically from the data.

E. Where’s My Line?
   A coordinate plane with 13 globs (see B-51), some 3 or 4 in colinear arrangements, is put on a transparency. Divide the class into teams of 2 or 3 students. Have a team select a particular set of colinear globs. Each team writes an equation for the line that fits the globs. Each team’s line (a separate transparency) is laid on the overhead by the teacher, based on the equation the team writes. A team’s score is based on the expression \(2^n-1\), where \(n\) is the number of globs which are on the line.

F. Find out what your yearbook publisher charges and set up some algebraic situations like the one that follows. Amy said the yearbook company will charge $6300 if 200 yearbooks are printed and $8900 if 400 yearbooks are printed. Write the equation that would describe this linear relationship.
G. Take advantage of data that appear in a newspaper or magazine. Here is an example. Between 1980 and 1990, the number of cable television subscribers increased about 3.6 million per year. In 1980 there were 17.5 million subscribers. Write a linear equation to estimate the number of cable subscribers. Check to see how well the equation estimates subscribers (see B-33).

H. Have students identify situations that may be modeled by linear equations if a slope and one point is known or if two points are known.

I. **Rise N’ Run**
   Students will play a game using graph paper and pick-up sticks. On a sheet of graph paper, two students draw the x- and y-axes, determine a scale for the axes, and number each axis. Place the graph paper on the floor. Begin the game by having the two students drop the sticks on the graph paper. Each student reads at least two points on the line made by their stick. Each student earns points by finding: the slope (1 point), the y-intercept (1 point), the line’s equation in slope-intercept form (2 points), the line’s equation in standard form, $Ax + By = C$ (3 points). Points are totaled and then summed with $A + B + C$ to decide the winner. A demonstration at the overhead should occur prior to letting the students work on their own. Students can use the calculator to compare their results with the location of the sticks.

J. **Connections** (W-5)

K. **Warm Ups** (W-36)

L. **Extra Essentials** (E-34, 35)

---

**Essentials for Instruction**

- Provide immediate feedback when possible.
- Use audiovisuals to introduce and/or summarize.
- Have students use logs or personal journals.
3.07 Investigate and determine the effects of changes in slope and intercepts on the graph and equation of a line.

a) Change only slope.

b) Change only the x- or y-intercept.

c) Change the slope and an intercept.

A. Develop an experiment to send a toy car down a ramp on various heights. Clock the time and determine the height of the ramp. Collect data and make graphs. Use the graphs to make generalizations about slope and intercepts.

B. Use a calculator to draw lines with the same slope but different intercepts on the same graph. Make generalizations. Do the same by using different intercepts, but the same slope.

C. Ask teams of two students to roll a die to determine the slope of a line. Then roll another die to determine the y-intercept. Draw the graph of the line. Repeat by rolling the dice again. (A negative sign can be used at any time.) Compare the graphs of the lines with the same slopes and same y-intercepts.

D. When given the general form of a linear, \( y = ax + b \), students should be able to discuss changes in the graph of the equation as \(|a|\) increases or decreases, changes in the graph when \(a\) is positive or negative, how \(a\) is related to the problem being addressed, changes in the graph of the equation as \(b\) increases or decreases, how \(b\) is related to the problem being addressed.

E. Graphing Linear Equations (A-43).

F. Moving on the Graph (B-45)

What do your students know about transforming a graph?

G. The Picture Tells the (Linear) Story (B-42, 43, 44)

Students investigate families of linear equations.

H. Connections (W-23, 53)

I. Calculator Tips (W-53)
3.08 Use linear equations or inequalities to solve problems. Solve by:

a) Graphing.

b) Using properties of equality; justify steps used.

A. Play Take a Bow. Give students cards with a “place” on the number line. Ask them to order themselves like a number line. Then direct the students to “take a bow” if their “place” is greater than 3, then less than \( \frac{1}{2} \), or greater than or equal to -5. Repeat with another group of students.

B. Use a relay format to practice solving inequalities.

C. Select an inequality (example: \( x - 5 < 2 \)). Enter each side of the inequality on the calculator (\( Y_1 = x - 5 \) and \( Y_2 = 2 \)). Graph, locate the intersection, and identify for which values of \( x \) is the left side above (greater) or below (less) the right side. Solve by hand and compare results with the calculator’s results.

D. Placekicker

In football, the place kicker can score points two different ways. He can score 3 points with a field goal or 1 point with a point-after-touchdown (PAT). The coach expects his kicker to score at least 50 points during the season. How many field goals and PATs could the kicker score?

E. Select an inequality (example: \(-2 < x + 7 < 6\)). Enter each part of the inequality on the calculator (\( y_1 = -2 \), \( y_2 = x + 7 \), and \( y_3 = 6 \)). Graph, locate the intersections, and identify for which values of \( x \) is the center part between the other two parts. Solve by hand and compare results with the calculator results.

F. Divide the students into teams. Have the students use any method they wish (including a calculator) to solve the equation. Keep score. Reward appropriately the team with the most correct solutions. Be sure to divide the teams equitably.

G. Re-Solving Equations (B-142, 143)

Students are given several simple linear equations that have been solved incorrectly. Students are expected to identify and explain errors and then solve the equation correctly. Whether we like it or not, students will generously supply teachers with new examples to share in this format. Use the blank setup (B-144) to adjust the exercises according to topics being studied.
H. Have students use the numbers 2, 3, and 5 (or any three of your choosing) in place of a, b, and c in the linear equation, \( ax + b = c \). How many different equations can be written? Have students record their equations, solve each equation, and compute: the probability that the solution is negative, the probability that the absolute value of the solution is 1, the probability that the solution is a fraction, and the probability that the solution is irrational.

I. Select a linear inequality. (Example: \( 4x + 2y \leq 6 \)) Solve for \( y \) in terms of \( x \). (\( y \leq -2x + 3 \)) Enter the corresponding linear equation in the calculator and graph. Identify the region above (greater) or below (less) the line as the solution set. Using the appropriate calculator function (Calculator Tips W-27, 33), graph and record. Choose several points in the region to check in the original inequality. Have the students try several inequalities.

J. **Estimating Fish Populations (A-29)**
Wildlife officials can approximate the number of fish in a pond to study the fish population and restocking needs. The method that they use is called the capture and recapture method. Students will use a simulation of this method and proportions to calculate a sample fish population.

K. Use algebra tiles to demonstrate solving simple equations using the additive inverse. Let students use tiles to solve similar problems generated from the teacher or textbook. For example:

\[
\begin{align*}
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ }
\end{array}
&= \\
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ }
\end{array}
\end{align*}
\]

\( 2x + 5 = x + 2 \)

Add -5 to each side of the equation and simplify.

\[
\begin{align*}
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{}
\end{array}
&= \\
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ }
\end{array}
\end{align*}
\]

\( 2x = x + -3 \)
Add $-x$ to each side of the equation and simplify.

$$\begin{array}{c}
\boxed{} & = & \boxed{}
\end{array}$$

$x = -3$

L. Use algebra tiles to demonstrate solving simple equations using the multiplicative inverse. Let students use tiles to solve similar problems generated from the teacher or textbook. For example:

$$\begin{array}{c}
\boxed{} & = & \boxed{}
\end{array}$$

$3x + 2 = x + 6$

Add $-2$ to each side of the equation and simplify.

$$\begin{array}{c}
\boxed{} & = & \boxed{}
\end{array}$$

$3x = x + 4$
Add \(-x\) to each side of the equation and simplify.

\[ 2x = 4 \]

Separate each side of the equation into two equal parts.

\[ x = 2 \]

M. Divide the students into teams. Have the students use any method they wish (including a calculator) to solve the equation. Keep score. Reward appropriately the team with the most correct solutions. Be sure to divide the teams equitably.

N. Solve an equation using paper and pencil. Students should identify properties, definitions, and procedures used at each step in the process. For example:

Solve for \(y\) in terms of \(x\):

\[
3(y + 2) - 7 = 3x - 3 \\
3(y + 2) - 7 + 7 = 3x - 3 + 7 \\
3(y + 2) = 3x + 4 \\
\frac{1}{3} \cdot 3(y + 2) = \frac{1}{3}(3x + 4) \\
y + 2 = x + \frac{4}{3} \\
y + 2 + -2 = x + \frac{4}{3} - 2 \\
y = x - \frac{2}{3}
\]

O. Have students use the numbers 2, 3, and 5 (or any three of your choosing) in place of \(a\), \(b\), and \(c\) in the linear equation, \(ax + b = c\). How many different equations can be written? Have students record their equations, solve each equation, and compute: the probability that the solution is negative, the probability that the absolute value of the solution is 1, the probability that the solution is a fraction, and the probability that the solution is irrational.
P. **Solve Equations Using Spreadsheets (A-45)**
Students will be given an equation. They will use values provided for \( x \) and enter these values into the spreadsheet along with both sides of the equation. After graphing the results as a line plot, students will see that the abscissa of the ordered pair at the intersection is the solution for \( x \) in the equation.

Q. Use algebra tiles to combine like terms and solve equations. Students should identify properties, definitions, and procedures used at each step in the process.

R. **Equation Relays**

- Divide the students into teams of three. Number each student in each team.
- Distribute the activity sheets (B-110, 111) to each team.
- Student #1 should write the first step for solving the first equation and then pass the sheet to student #2. Student #2 should write the next step for solving the equation. The sheet should continue to be passed until the equation is solved.
- For the next equation have student #2 start the process, and so on.
- Continue until all ten equations are solved.

S. Ask the students to bring in a set of data from the newspaper or a magazine. In groups, develop problems from the data. Find the solutions using any appropriate method. Have pairs of students report on their findings.

T. Use the puzzle formats of **How Do They Fit? (A-15), Lining Up Dominoes (A-1), “I Have ... Who Has ...” (A-3), and Relays (B-25, 26)** to create puzzles for student use. Whenever possible, let students create the puzzles. These formats can also be used to address evaluating algebraic expressions, simplifying real number expressions, raising a number to a power, simplifying radical expressions, multiplying binomials which contain roots, solving a variety of equations and inequalities, and operating with polynomials.

U. Use the game formats outlined on E-21, 22, 23 to practice solving a variety of equations and inequalities. Encourage students to create their own versions of popular (legal, of course) games.

V. Construct linear equations and inequalities to solve problems related to the Income Tax Rate Schedules (B-74). See B-75 for some examples.

W. **Connections** (W-5, 23, 57)

X. **Calculator Tips** (W-21, 23, 53)

Y. **Warm Ups** (W-18, 20, 24, 26, 28, 32)

Z. **Challenges** (W-54)

AA. **Extra Essentials** (E-25, 32, 34, 35, 36)
3.09 Use systems of linear equations or inequalities in two variables to solve problems. Determine the solution by:

a) Graphing.

b) Substitution.

c) Elimination.

A. Olympic Swimming (B-53)

With the Olympics occurring every two years (alternating summer and winter games), there are several events in which both men and women compete. For instance, there are winning results for men and women in the 400 meter free-style swimming since 1924. Working in pairs, students will select (or be assigned) an event. Using their calculators, they would determine best-fit linear equations for each of the men’s and women’s data. If appropriate, use the equations to determine men’s and women’s performances for 1940 and 1944. (Why were there no results those years?) Predict the winning results for the next several Olympics. Ask the students to determine, according to their calculations, if the women’s performance will ever equal or exceed the men’s performance in their event. Research as to whether this is likely to happen. At the 2004 Athens Olympics the winning times in the 400 meter free-style events were 223.10 seconds for the men and 245.34 seconds for the women.

B. Have students research the transportation costs for travel between cities. Assume that the costs identified represent a linear trend. Here is an example (01/11/03).

<table>
<thead>
<tr>
<th>distance (round trip)</th>
<th>car</th>
<th>air</th>
<th>rail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raleigh-Charlotte</td>
<td>300 miles</td>
<td>$108</td>
<td>$224</td>
</tr>
<tr>
<td>Raleigh-New York</td>
<td>1300 miles</td>
<td>$468</td>
<td>$169</td>
</tr>
</tbody>
</table>

Determine the distance at which driving a car is less expensive than riding the train. When does it become cheaper to fly rather than drive? For what distance is the train the most expensive mode of travel? Identify some advantages and disadvantages for each mode of transportation. What other variables affect the cost of travel?

C. Select two linear equations and enter them on the calculator. (Example: \( Y_1 = x - 4 \) and \( Y_2 = -x + 3 \))

Graph, trace, and identify the intersection. Verify with substitution. Record the graph and coordinates of the intersection. Have students try other pairs of equations and record results. Use a friendly range for best results.
D. During the 1999 baseball season, each team in Major League Baseball played 162 games. On April 30, the Boston Red Sox had played 22 games and won 11. Meanwhile the New York Yankees had won 14 of their 21 games. By May 31, Boston had played 50 games and won 31. New York had played 48 games and won 28. Create a linear equation in slope-intercept form that describes the trend for each team. Define the slope of your equations with respect to the quantities being discussed. Explain how you created your equations and use them to predict the number of games each team should win by the end of the season. How do your results compare with the actual results for that season? Explain the difference, if any.

E. Divide students into pairs. Give each pair two systems to graph. Student A graphs the first system at the same time Student B graphs the second system. Have the pair exchange papers to find the solution to the systems from the graph and check the solutions. Each pair works together if difficulties arise.

F. Divide class into cooperative groups. Give each group three graphs of systems of equations and the tables from a spreadsheet. Have groups write the equations for each system, find the solutions from the graphs, and check the solutions using the equations.

G. Select two linear inequalities and enter the corresponding linear equations on the calculator. Using the appropriate calculator function (Calculator Tips, W-21, 23), graph and identify the intersection. Record the graph. Have students try other pairs of inequalities and record results.

H. Survey students and create problems similar to the one that follows. Mildred makes $3 an hour babysitting and $6 an hour when she works at Wendy’s. Her parents do not want her to work more than 20 hours per week. Mildred would like to earn at least $70 a week. Write a system of inequalities that show the number of hours she could work at each job. Graph the system. Write at least four possible solutions.

I. Use information from local businesses to create problems. Here is an example. The Twin Theater charges $7 for adult tickets and $4 for children 12 or under. The theater has 470 seats. The manager wants to have a nightly income (two shows) of at least $5000. Write a system of inequalities for the number of children and adult tickets that can be sold. Write at least four possible solutions.

J. Have students write a system of inequalities whose solution set is (1) a triangle, (2) a trapezoid, (3) a kite, (4) a hexagon.

K. A business will make money if revenues exceed expenses. Discuss with students the break-even point when revenues equal expenses. Have students consider the following hamburger business. The owner pays $20,000 for the franchise and has expenses of $750 per thousand hamburgers. The price of a hamburger is $2.09. Have students write an equation to represent the cost of the business and the equation to represent the revenue. Graph the equations. Ask the students to find the number of hamburgers the business needs to sell to break even. What would be the profit if 100 thousand burgers are sold? Many franchises work this way. Have students talk with local owners and share the information with the class.
L. Use business-type situations that arise at school to create problems. Here is an example. The Silver Ratio Band wants to talk to the school principal concerning a contract to play for the Valentine dance. The group is considering three possible rates: (1) the band will charge $3 per person; (2) the school will pay the band $50 plus $2.50 per person; (3) the band will rent the fellowship hall at a local church for $125 and charge $4 per ticket. Which method would be best for the group to use? Have students write a summary of their findings.

M. Ask students to describe the advantages of each method of solving systems of equations: graphing a system of equations on a calculator and using a spreadsheet to analyze a system of equations.

N. I’m thinking of two numbers. Their sum is 12. The sum of the first with twice the second is 7. What are the numbers? (17, -5)
I’m thinking of two numbers. Their difference is 9. The sum of twice the first and three times the second is 63. What are the numbers? (18, 9)
I’m thinking of two numbers. The first is three times as large as the second. Their sum is 48. What are the numbers? (36, 12)
Ask students to create their own versions of “I’m thinking of two numbers.” Collect those from the students, compile and edit, and redistribute for students to solve.

O. Give each row of students one of the following sets of equations to solve using the addition method: (1) $x + 2y = 3, 4x + 5y = 6$; (2) $2x + 3y = 4, 5x + 6y = 7$; (3) $3x + 4y = 5, 6x + 7y = 8$; (4) $4x + 5y = 6, 7x + 8y = 9$; (5) $5x + 6y = 7, 8x + 9y = 10$. Call on one person on each row to give the solution. Ask students if they notice a pattern to the equations. Why would they have the same solutions? If all systems were graphed on the same axis, what would they look like?

P. Calculator Wars
(1) Each player enters the equations for two lines into $Y1$ and $Y2$. Wise players choose equations that do not intersect on the screen.
(2) After they exchange calculators, the students must manipulate the window so that the crossing lines appear. They must then use the 2nd CALC INTERSECTION to find the solution to the system of equations. Since the calculation will only find the intersection of the functions displayed on the screen, the students may not skip the step of manipulating windows.
(3) Encourage the players to use the table function to get initial estimates.
(4) The first player to show his opponent the correct solution wins the round. As always, play continues until the teacher calls time.

Q. First Wheels (B-54)
Construct a system of equations to make a thoughtful automobile purchase.
R. Students can use linear inequalities to solve a geometric probability problem.

Sandy proposes the following problem to her friend, Mira. Suppose you choose any two numbers less than 180, what is the probability that the two numbers can be measures of angles in a triangle?

Experimental Probability: Simulate the experiment by randomly generating a sample of 40 pairs of random numbers less than 180. (use \texttt{iPart 180rand + 1} on the TI-82 to generate a random number less than or equal to 180. Use \texttt{randInt(1,180,2)} on the TI-83.) Look at the pairs and calculate the probability that the two numbers are less than 180.

Theoretical Probability: Let \( x \) = the measure of \( \angle 1 \) and \( y \) = the measure of \( \angle 2 \). Write three inequalities about these measures in relation to 180. (\( x < 180 \), \( y < 180 \), and \( x + y < 180 \)).

Draw a graph to represent all possible values for \( \angle 1 \) and \( \angle 2 \). Shade the inequalities that can be values of the angles of a triangle.

What is the area of the geometric shape representing the values of angles of a triangle?

Josh proposes the following change. What if we use any numbers less than 200 for the measures of \( \angle 1 \) and \( \angle 2 \). What is the probability that the numbers can be measures of angles in a triangle?

a. Simulate the experiment using 40 pairs of random numbers.
b. Draw the sample space of all possible numbers that are used.
c. Shade in the numbers that can be angles in a triangle.
d. Calculate the areas for the probability. (Answer: 0.405)

Maria proposes one more problem. Let’s use only numbers less than 100 for \( \angle 1 \) and \( \angle 2 \). What is the probability if you choose two numbers at random that the two numbers can be angles in a triangle?

a. Simulate the experiment using 40 pairs of random numbers.
b. Draw the sample space for all possible numbers used.
c. Shade in the numbers that can be angles in a triangle.
d. Calculate the areas for the probability. (Answer 0.8)

S. Revisit absolute value equations using the substitution method and calculators. Have students graph on the calculator, \( y = |x - 2| \) and \( y = 4 \). Use the trace to find the ordered pairs for the intersection. Next have students solve the system using substitution. (i.e. \( |x - 2| = 4 \))

Ask students to explain a method for solving an absolute value equation, \( |x - 3| = 8 \) using a calculator and solving by hand.

T. Warm Ups (W-16)

U. Challenges (W-10, 40, 46, 54, 56, 60)

V. Extra Essentials (E-32, 33, 36, 37, 38, 43)
3.10 Graph quadratic functions.
   a) Locate the intercepts and the vertex.
   b) Recognize the x-intercepts of the function as the solutions of the equation.

A. Have students hand-graph examples of quadratic equations that are in the textbook. Pair students and ask them to generate a list of characteristics of each equation and quadratics in general.

B. Have students graph \( y = 2x - 1 \) and \( y = x^2 - 1 \) on separate graphs. Ask them to write a description of how the graphs are alike and how they are different.

C. The Picture Tells the (Quadratic) Story (B-76, 77). Students investigate families of quadratic equations.

D. Have students use and investigate quadratic equations that could represent physical phenomena. The quadratic equation \( y = -4.9x^2 + 30x + 1.5 \) describes in meters the height of a baseball that is traveling at 30 meters per second and has been hit from a height of 1.5 meters above the ground. (1) Make a table of values, (2) graph the equation, (3) as time increases, describe the height of the ball, and (4) how does the graph differ from a linear equation?

E. Have students graph some quadratic equations which model a situation they may be familiar with. (B-78).

F. Max-Min
   Give students three points to graph on graph paper. Have them sketch in a parabola that includes the three points and estimate the maximum or minimum value for the parabola. Have the students share their results. After several examples, give students two points to work with. Again, have students share their results. How many points are needed to determine a parabola?

G. Open Boxes (A-49).
   In groups, students will collect and analyze data from the construction of several boxes and estimate the maximum volume. Students will use an algebraic model to determine a maximum volume and compare it with the experimental results.

H. The Maximum Garden (A-51).
   Students will use a table to list possible values for the dimensions and area of a garden space. Students will write an equation to graph the width of the garden verses the area. This will be a quadratic function with a maximum value.

Resources for Algebra •• G-45 •• Public Schools of North Carolina
I. Have students relate the solutions of quadratic equations to the graphs of related functions.
   (1) Solve by factoring (a) \( x^2 + x - 6 = 0 \)  (b) \( x^2 - 8x + 12 = 0 \)
   (2) Now graph on the calculator the related functions and record the graphs. Where does each function cross the \( x \)-axis? (a) \( y = x^2 + x - 6 \)  (b) \( y = x^2 - 8x + 12 \)
   (3) How can the solutions to quadratic equations be found by graphing the related function? Why?
   (4) Find the solutions to the nearest tenth.
   (a) \( x^2 + 2x - 7 = 0 \)  (b) \( 2x^2 - 8x + 3 = 0 \)  (c) \( 4x^2 = 8x + 7 \)

J. Make an overhead of a picture of a dolphin jumping. Have students graph on a graphing calculator this equation, \( y = -0.35x^2 + 1.48x \) where \( x \) is the horizontal distance of a dolphin jump and \( y \) is the vertical distance of the jump in meters. Ask students to find how high the dolphin jumps and after how many meters will the dolphin enter the water.

K. Quadratic Functions (A-47).
   Students will use the quadratic formula to obtain a solution, in radical form, to the related quadratic equation. Using a calculator, they will convert the solutions to decimals, rounding to tenths. Students will then graph the quadratic function on a calculator. They will use the zoom and trace keys to estimate the \( x \)-intercepts of the function to the nearest tenth (or they may use the table function).

<table>
<thead>
<tr>
<th>Essentials for Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use short answers whenever possible.</td>
</tr>
<tr>
<td>Have students repeat directions.</td>
</tr>
<tr>
<td>Select materials relevant to students.</td>
</tr>
<tr>
<td>Visually illustrate new vocabulary.</td>
</tr>
</tbody>
</table>
3.11 Use quadratic equations to solve problems.
Solve by:
a) Factoring.
b) Locating points on the graph.

A. Shuttle Launch (A-53).
Working in pairs, students will use a pair of quadratic equations to identify the critical points along the flight of the solid rocket boosters (SRB) that are used to launch the space shuttle. Students will connect algebraic ideas (intersection, vertex, x-intercept, and evaluating expressions) with points along the flight path (engine shutdown, maximum altitude, splashdown, and altitude verses elapsed time).

B. Use quadratic equations to represent a commercial situation. The profit of a business can be described by the equation \( P = 1.8T^2 - 20T + 250 \), where \( P \) is the profit in thousands of dollars and \( t \) is the number of years since 1995 (\( t = 0 \) corresponds to 1995). Describe the profit trend over the last ten years. Use the model to predict in what year the profit will be double that of 1995.

C. Have students solve equations in factored form with a product of zero from the textbook. Have them explain (1) why does this method work, (2) how can you tell if a solution will be positive or negative, (3) how can you tell if a solution will be non-integral.

D. Have the students reverse the processes. Ask them to write a quadratic equation that has (1) two positive solutions, (2) two negative solutions, (3) two solutions that are fractions, (4) has only one solution.

E. Have students discuss why the following is incorrect. Tanya solves a quadratic equation this way.
\[
(2x - 1)(x + 2) = 3 \\
2x - 1 = 3 \quad \text{or} \quad x + 2 = 3 \\
x = 2 \quad \text{or} \quad x = 1
\]
Does \( x = 2 \) check as a solution to the equation? Does \( x = 1 \) check as a solution? What is wrong with Tanya’s method?

Students will fill in the entries for \( Y_1 \) and \( Y_2 \) with binomials and, using calculators, determine and record the graphs of the products of the binomials. Students are expected to identify the solutions (x-intercepts) of linear and quadratic equations for each graph in the matrix. Students can use a similar process to explore the sums, differences, products, and quotients of varying degrees of polynomials.
G. **Factoring Trinomials with Algebra Tiles (A-37).**
Working in pairs, students will select algebra tiles corresponding to the terms of a given quadratic trinomial. The students will create a rectangular arrangement with the tiles and identify the dimensions of the rectangle. Each dimension will be one of the algebraic factors of the original trinomial.

H. **Polynomial Four-in-a-Row (B-60)**
Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until someone gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another. A blank game board for **Four in a Row** is provided (B-61) so that teachers can give students the opportunity to create their own versions and address specifically other objectives in algebra.

I. Use the matrix method for factoring quadratic expressions. See 1.03E.

J. Have students solve second-degree equations by factoring using problems from their textbook. Ask (1) will the method always work and (2) write a second-degree equation that cannot be factored.

K. Give students a table of values for \( y = x^2 - 2x - 8 \).

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7</td>
<td>0</td>
<td>-5</td>
<td>-8</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

(1) From the table, find solutions when \( y = -5 \) or \( x^2 - 2x - 8 = -5 \).
(2) Now use factoring to find the solutions.
(3) Solve each of the following using the table, then factoring.
   (a) \( x^2 - 2x - 8 = 7 \)
   (b) \( x^2 - 2x - 8 = -8 \)
   (c) \( x^2 - 2x - 8 = 0 \)
(4) Develop a table with ten ordered pairs using the function \( y = x^2 + 2x - 15 \). Write four equations you can solve using your table. Solve the four equations by factoring.

L. Use the cover-up method to have students solve equations such as \( (x + 2)^2 = 49 \). On the overhead, write \( (x + 2)^2 = 49 \) and place a square of paper over the term inside of the parentheses.
\( (\phantom{x+2})^2 = 49 \). So \( x + 2 = 7 \) or \( x + 2 = -7 \) which means \( x = 5 \) or \( x = -9 \).
M. Use an application. Each side of a square patio was increased in length by 5 feet to give it an area of 150 square feet. What was the original length of the patio? If $x$ is the original length, solve for $x$ when $(x + 5)^2 = 150$.

N. With the calculator, have the students trace and zoom to find solutions for equations set equal to zero. (Example: $x^2 - 3x + 4 = 0$)

O. When given equations like $x^2 - 3x + 4 = 7$, have students enter each side of the equation in the calculator as $Y_1 = x^2 - 3x + 4$ and $Y_2 = 7$. Graph, trace, and zoom to locate the intersection.

P. Using the North Carolina population information, B-52, predict the population in 2000. In what year should the population of North Carolina reach eight million? Try the quadratic curve of best fit with the calculator.

Q. **Calculator Tips** (W-67, 71)

R. **Connections** (W-29)

S. **Warm Ups** (W-72)

T. **Extra Essentials** (E-25, 26, 35)

---

### Essentials for Instruction

- Underline or highlight important words in directions or test items.
- Reduce copying from the chalkboard or overhead.
- Provide a list of all assignments given.
- Have frequent review.
3.12 Use formulas and graphs to solve problems involving exponential functions.
Solve a problem by:
a) Locating points on the graph.
b) Evaluating an exponential expression.

A. The Dinosaurs Bite the Dust (B-83)
Simulate the extinction of the dinosaurs using dice to create an exponential model. Point out to students the random nature of the dinosaurs’ demise, yet we are able to discover a mathematical model for the event. The two theories which try to explain the dinosaurs’ disappearance are provided only to the teacher. The teacher can share this information with the students or expect the students to research the topic.

Sixty-five million years ago Earth experienced a global extinction event so severe that it defines the boundary between the Cretaceous (K) and Tertiary (T) geological periods. Causing extinctions on both the lands and in the oceans, that event is referred to as the K-T extinctions. The dinosaurs became extinct during the K-T extinctions.

Of all the theories ever devised for cause of the K-T extinctions, only two remain and they are the focus of intense scientific debate. One, the asteroid-impact winter theory created by Nobel laureate Luis Alvarez, states that a giant asteroid struck Earth 65 million years ago. It blasted dust into the stratosphere that blocked out sunlight and plunged Earth into a dark, frozen winter.

The other, the volcano-greenhouse theory originated by Dewey M. McLean, relates the K-T extinctions to a major perturbation of earth’s carbon cycle caused by the Deccan Traps Mantle Plume Volcanism in India. This was one of the greatest volcanic events in Earth history and its main eruptions began 65 million years ago. The Deccan Traps released vast quantities of the greenhouse gas, carbon dioxide (CO₂), onto Earth’s surface, trapping heat from the sun, and turning Earth’s surface into a hot, sterilizing “greenhouse.”

It has been estimated that the dinosaurs disappeared in 100-300 years. Using the function \( y = 500000 \cdot (G)^x \), where \( x \) is the number of years since the extinction event, \( y \) is the remaining dinosaurs, and \( G (.01 \leq G \leq .99) \) is the rate of population decline, have students determine a rate of decline when there are 100 dinosaurs left after 300 years.
B. **Problems of an Exponential Nature (B-84)**

Many problems like these are available in textbooks. Expect students first to investigate the situation in a table, using their calculators for computation. Then students can discover and discuss patterns in a graph of their data. Students can explore curve-fitting with their calculators and/or discuss how to use the \( y = a \cdot b^x \) form to model the problem.

C. **Patterns with Exponential Equations (A-55)**

Students will graph equations in which the base \( b \) is a positive number greater than 1. They will investigate what happens as \( b \) increases and describe the pattern. Next, students will graph equations in which the base \( b \) is between 0 and 1 and describe this pattern.

D. **Rolling Dice: An Exponential Experience (B-80)**

Begin with a large number of dice (30+). Place the dice in a cup and roll them. Remove all the dice that show 3. Roll the remaining dice and again remove the 3s. Continue the process until there are only one or two dice remaining. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( y = N \cdot \left(\frac{5}{6}\right)^x \), where \( N \) is the number of dice you begin with, \( x \) is the number of rolls, and \( y \) is the remaining dice?

With a large number of dice (30+) handy, begin with two dice. Place the dice in a cup and roll them. For every die that shows a 3, add another die. Roll the dice and again add a die for each 3 that appears. Continue the process until all of the dice are used. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( y = N \cdot \left(\frac{7}{6}\right)^x \), where \( N \) is the number of dice you begin with, \( x \) is the number of rolls, and \( y \) is the new dice total?

E. Begin with a large number of coins (30+). Place the coins in a cup, shake, and dump on the desk top or floor. Remove all the coins that show HEADs. Shake and dump the remaining coins and again remove the HEADs. Continue the process until there are no coins remaining. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( Y = N \cdot \left(\frac{1}{2}\right)^x \), where \( N \) is the number of coins you begin with, \( x \) is the number of rolls, and \( y \) is remaining coins?

With a large number of coins (30+) handy, begin with two coins. Place the coins in a cup, shake, and dump on the desk top or floor. For every coin that shows HEADs, add another coin. Shake and dump the coins and again add a coin for each HEADs that appears. Continue the process until all of the coins are used. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( Y = N \cdot \left(\frac{3}{2}\right)^x \), where \( N \) is the number of coins you begin with, \( x \) is the number of turns, and \( y \) is the new coin total?
F. **Use Your Imagination (B-82)**
Three problems are presented that are mental experiments involving “folding” a single sheet of paper in half many times. These are good illustrations of the power of exponential growth. The solutions are the Sears Tower in 23 folds, Mount Everest in 27 folds, and the Moon in 42 folds.

G. Have students use a calculator and a table to calculate the cost of jeans at 5% inflation over a period of 5 years. Then, have them compare their yearly results to the compound interest formula.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost</th>
<th>30(1.05)^0 = 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>30(1.05)^1 = 31.50</td>
</tr>
<tr>
<td>1</td>
<td>30 + .05\times30 = 31.50</td>
<td>30(1.05)^2 = 33.08</td>
</tr>
<tr>
<td>2</td>
<td>31.50 + .05\times31.50 = 33.08</td>
<td>30(1.05)^3 = 34.73</td>
</tr>
<tr>
<td>3</td>
<td>33.08 + .05\times33.08 = 34.73</td>
<td>30(1.05)^4 = 36.47</td>
</tr>
<tr>
<td>4</td>
<td>34.73 + .05\times34.73 = 36.47</td>
<td></td>
</tr>
</tbody>
</table>

Write an expression for the cost of jeans in the year 2010. Use your calculator to simplify.

H. Have students draw a graph to compare the difference in simple interest and compound interest for $2,000 invested at 6%.

<table>
<thead>
<tr>
<th>Years</th>
<th>Simple Interest</th>
<th>Compound Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000\times.06\times0+2000=</td>
<td>2000\times1.06^0</td>
</tr>
<tr>
<td>10</td>
<td>2000\times.06\times10+2000=</td>
<td>2000\times1.06^{10}</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I. **Calculator Tips (W-61, 63)**

J. **Connections (W-69)**

K. **Warm Ups (W-70)**

L. **Challenges (W-72)**

M. **Extra Essentials (E-25, 26, 27, 30)**
### Data, Probability, and Statistics

#### 4.01 Use matrices to display and interpret data.

**A. National League Matrix (1997 season through July 13, from *USA Today*)**

The rows (top to bottom) and columns (left to right) are arranged alphabetically by teams: Atlanta, Chicago, Cincinnati, Colorado, Florida, Houston, Los Angeles, Montreal, New York, Philadelphia, Pittsburgh, St. Louis, San Diego, San Francisco. A team’s number of victories against each opponent can be found by reading across. The number of losses can be found by reading down. What is Atlanta’s overall record against the other National League teams?

<table>
<thead>
<tr>
<th></th>
<th>At</th>
<th>Ch</th>
<th>Ci</th>
<th>Co</th>
<th>Fl</th>
<th>Ho</th>
<th>Mo</th>
<th>NY</th>
<th>Ph</th>
<th>Pi</th>
<th>St</th>
<th>SD</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>At</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>8</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Ch</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Ci</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Co</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Fl</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Ho</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>LA</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Mo</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>NY</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ph</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Pi</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>St</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>SD</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**B. Use the NHL standings from the newspaper to create a matrix that reports points (based on wins and ties), goals scored, and goals allowed. Based on the matrix, how do points correlate with goals scored? Goals allowed? If a team had 25 wins, how many goals would it be expected to score? Allow?**

**C. An almanac is a great source of data to use in matrix formats. For instance, use the Average Daily Temperatures in Tourist Cities from the *Information Please Almanac* to create matrices for North America, Europe, and the Southern Hemisphere. Compare temperature variations among the cities. Compare the data between Europe and the Southern Hemisphere; differentiate between the two sets. Compare data between cities close to the equator and those not. Find similar information about major US cities and report in a matrix. Do the same for NC cities.**

*Resources for Algebra •• G-53 •• Public Schools of North Carolina*
D. Use the matrix which accompanies the NC maps published by the NC Department of Transportation to determine distances between NC cities. Use the matrix to estimate driving distance for a trip to the NC mountains during Leaf Season. One such trip may follow this route for a round trip from Raleigh. Raleigh – Greensboro – Charlotte – (via US 74 and I-26) - Asheville – Boone – Winston-Salem – Raleigh.

E. Create and update a matrix for ACC basketball, through the ACC tournament, similar to the one for major league baseball.

F. Use the tax tables in the current IRS 1040 Forms and Instructions or NC Individual Income Tax Forms and Instructions tax materials to determine taxes owed for varying amounts of taxable income. Describe how the tax tables are organized (income span per row, relationships among filing categories, how does the tax owed change as the income increases, identify similarities and differences between the state and federal tables, etc.). Compare the tables with the actual tax rate schedules for both forms.

G. Have students bring in prices of a large pizza with one topping, large drink, and extra topping from three or four favorite pizza restaurants. Set up a 3 • 4 matrix. Here is one example:

<table>
<thead>
<tr>
<th>Italian Delight</th>
<th>Dot’s House</th>
<th>Pizza Romans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Pizza</td>
<td>$8.99</td>
<td>$11.40</td>
</tr>
<tr>
<td>Large Drink</td>
<td>.99</td>
<td>.85</td>
</tr>
<tr>
<td>Extra Topping</td>
<td>1.30</td>
<td>1.25</td>
</tr>
</tbody>
</table>

H. Have students investigate information from two matrices. For example: Pete investigates interest rates at First Bank and Credit Savings for certificates of deposit (CD) and money market accounts. This information is listed in the 2 • 3 matrix.

<table>
<thead>
<tr>
<th>12 CD</th>
<th>48 CD</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Bank</td>
<td>5.8%</td>
<td>6.8%</td>
</tr>
<tr>
<td>CreditSav</td>
<td>6.5%</td>
<td>6.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12 CD</th>
<th>48 CD</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option I</td>
<td>$15,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>Option II</td>
<td>$10,000</td>
<td>$25,000</td>
</tr>
</tbody>
</table>

What would be the interest for one year on a 48 month CD for option I at First Bank?
What would be the total interest for option I at Credit Savings?
Now compare this to option I at First Bank?
Compare the total interest for option II at both banks.
Which investment option would be the best for Pete?

I. Using the matrix for In-Line Skating (B-33), add a row that estimates calories consumed for a 130 pound skater. As speed increases how do the number of calories consumed at each weight change? Do the calories increase at the same rate at every weight?
Using the matrix for Personal Fitness (B-34), add a row that estimates calories consumed by a person running 13 minute miles. For a person weighing 130 pounds, how many calories does he consume in each activity?
J. Determine the top 15 countries that earned gold, silver, and bronze medals at the 1996 Olympics. Organize a matrix to display the information. Order the data so that there are columns for gold, silver, and bronze medals won. Order the rows according to the country that won the most gold medals at the top of the matrix and the country that won the least gold medals at the bottom. If two or more countries had the same number of gold medals, use the most silver medals, and then bronze if needed. Add a fourth column with the medal totals. The matrix can be expanded by adding a column that shows the most recent census or population estimate of each country. In another column record how many medals were won per thousand people in each country. Analyze and discuss any trends or relationships that you notice in the matrix.

K. Have students set up a matrix of sports data for their favorite team (see B-17 for an example) on a spreadsheet. Have them enter at least three formulas to calculate averages or totals for the players or the team. How could the matrix (B-17) be organized to better investigate correlations between minutes played and points scored or free throw attempts and rebounds? If a player had 1700 minutes of playing time, how many points should he have been expected to score? If a player collected 250 total rebounds, how many free throws would I have expected him to attempt?

L. Have students set up a matrix of data on the planets of our solar system on a spreadsheet. Enter columns for equatorial diameter, mean solar distance, and orbital period. Use the spreadsheet to create new columns in the matrix which compare planetary diameters with Earth’s and mean solar distance in astronomical units. Look for patterns in the data and potential correlations (mean solar distance and orbital period) by reordering rows according to mean solar distance, equatorial diameter, etc.

M. **Connections** (W-3, 11, 15, 17, 35, 43, 49, 51, 61, 63, 67, 71)

| Essentials for Instruction | Vary test format (written, oral, short answer, essay, multiple choice, true-false, matching, computation, yes-no, performance testing).

Use a grading system that reflects the varied activities of mathematics instruction.

Provide feedback to parents.
4.02 Recognize and identify linear and non-linear data.

A. **Teacher to Teacher** (W-51) identifies several sources for useful data. See B-33, 34, and 79 for some examples. Give students opportunities to research and select data, graph them, and determine whether they behave in a linear fashion. Determine lines of best fit and make predictions within the context of the data. See Scoring and Winning (A-7), The Wave (A-17), and How Do You Measure Up? (A-19) for examples. Compile the data sets and with modifications use for homework, starter problems, quiz items, and test items.

B. In groups, have students draw two graphs of data that represent linear relationships and two that are not linear. Groups must explain graphs to the class.

C. Give students tables of values that may or may not be linear. Have them investigate by graphing the data which are linear. Ask if they can find a method by just observing the tables to determine whether the data behave in a linear fashion.

D. This activity can be done in a computer lab with tables of data already on a saved spreadsheet. Have students open the spreadsheet, graph the tables, and record which produce linear graphs. Have students determine what type of table will always be a linear graph.

E. Show pictures of graphs on the overhead. Ask which are linear.

F. **Patterns in Perimeter** (B-35, 36, 37, 38) and **Patterns in Area and Volume** (B-39, 40, 41).

G. **Rolling Dice: An Exponential Experience** (B-80)
   
   Begin with a large number of dice (30+). Place the dice in a cup and roll them. Remove all the dice that show 3. Roll the remaining dice and again remove the 3s. Continue the process until there are only one or two dice remaining. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( y = N \cdot \left(\frac{5}{6}\right)^x \), where \( N \) is the number of dice you begin with, \( x \) is the number of rolls, and \( y \) is remaining dice?

With a large number of dice (30+) handy, begin with two dice. Place the dice in a cup and roll them. For every die that shows a 3, add another die. Roll the dice and again add a die for each 3 that appears. Continue the process until all of the dice are used. Keep a record of the results for each roll and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( y = N \cdot \left(\frac{7}{6}\right)^x \), where \( N \) is the number of dice you begin with, \( x \) is the number of rolls, and \( y \) is the new dice total?
H. Begin with a large number of coins (30+). Place the coins in a cup, shake, and dump on the desk top or floor. Remove all the coins that show HEADs. Shake and dump the remaining coins and again remove the HEADs. Continue the process until there are no coins remaining. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( Y = N \cdot \left( \frac{1}{2} \right)^x \), where \( N \) is the number of coins you begin with, \( x \) is the number of rolls, and \( y \) is remaining coins?

With a large number of coins (30+) handy, begin with two coins. Place the coins in a cup, shake, and dump on the desk top or floor. For every coin that shows HEADs, add another coin. Shake and dump the coins and again add a coin for each HEADs that appears. Continue the process until all of the coins are used. Keep a record of the results for each turn and graph those results. Use the calculator to determine a best-fit exponential function. How does the best-fit function compare with the expected function \( Y = N \cdot \left( \frac{3}{2} \right)^x \), where \( N \) is the number of coins you begin with, \( x \) is the number of turns, and \( y \) is the new coin total?

I. Have students compare tables of values for a variety of equations. (B-81).

J. Give students tables of values and ask them which represents an exponential equation. (B-81)

K. Give students pictures of graphs and ask them which represents exponential data. (B-81)

L. **Use Your Imagination (B-82)**
   Three problems are presented that are mental experiments involving “folding” a single sheet of paper in half many times. These are good illustrations of the power of exponential growth. The solutions are the Sears Tower in 23 folds, Mount Everest in 27 folds, and the Moon in 42 folds.

M. Using the North Carolina population information, B-52, predict the population in 2000. In what year should the population of North Carolina reach eight million?

N. CBLs and similar electronic devices should be used whenever possible as generators of “real world” data for students’ inspection and analysis. How Hot (Cold) Is It? (A-11) is one such activity. Although the only curve-of-best-fit expected in Algebra I is a linear one, should sets or subsets of data collected be exponential or quadratic in nature, it would be appropriate for students to try the other best-fit utilities of their calculators.

O. **Calculator Tips** (W-27, 29)

P. **Connections** (W-3, 11, 15, 17, 27, 31, 35, 49, 51, 61, 65, 67, 71)

Q. **Warm Ups** (W-40)

R. **Extra Essentials** (E-24)
4.03 Create and use linear models based on real data.
   a) Graph the data.
   b) Write a linear equation which models a set of real data.
   c) Describe the slope and intercepts in the context of the data.
   d) Check the model for goodness-of-fit and use the model to make predictions.

A. Have students graph data from an almanac or other source. Teacher to Teacher (W-51) identifies several sources for useful data, see B-33, 34 for some examples. Write an equation to model the relationship.

B. Take advantage of data that appear in a newspaper or magazine. Here is an example. In 1986, the 20th Super Bowl was played in New Orleans. The price of a 30-second television commercial for the game was $550,000. In 1993, the 27th Super Bowl was played in Pasadena and the price of a 30-second commercial had risen to $850,000. Write a linear equation that models the change in the price of television commercials for this sporting event.

C. Scoring and Winning (A-7)
   Have students gather data from the NFL (or similar data from the NBA, NHL, MLB, or local minor leagues) to create scatter plots and find lines of best fit. Students will discuss the characteristics of those lines and make predictions.

D. Find the flight times and direct air distances from the larger North Carolina airport (Charlotte, RDU, Piedmont Triad, etc.) near you. Use airline tables and be aware of time zone considerations as flight times and air distances are compiled. Plot the data, find the line (or other curve) of best fit. Discuss the meaning of slope and x- and y-intercepts in the context of the data.

E. The Good Estimator (A-9)
   Individual students or groups will estimate ten distances in the classroom or other parts of the school. Then the students will measure the same distances and compare their estimates with the “real” measures.
F. **The Wave (A-17)**
   In a whole class setting, increasingly larger groups of students will perform the “wave”. The students will collect and interpret data, determine a linear function of the time to complete the “wave” dependent upon the number of students participating, and use the linear function to make predictions.

G. **Patterns in Perimeter (B-35, 36, 37, 38)**
   Students will use perimeter to generate data, find the algebraic expression for the linear pattern, and graph the data that are generated. Although it is expected that students will work individually or in pairs on the pattern sheets, overhead versions of two of the problems are provided for whole class discussion. Using pattern blocks to build the figures is particularly helpful for many students’ understanding.

H. After trying **Basketball: With the game on the line ... (A-31)**, try **Basketball Extension 1 (B-15)**. The **Basketball** activities generally connect probability and statistics with algebra.

I. **Used Cars**
   Have students ask 10-12 teachers the ages and mileages for their cars. Have the class draw a scatter plot and a line of best-fit. Ask students to estimate a slope and interpret the slope in the context of the situation.

J. Give the equation \( y = 3.5x + 25 \), have students create a situation modeled by the equation. For instance, let the equation describe the relationship between the number of martial arts classes \( x \) and their cost \( y \) at the YMCA. What does the y-intercept represent? What does the slope represent?

K. **Connecting Units of Measure (A-25)**
   Students will measure several objects in the classroom using both centimeters and inches. Students will plot corresponding pairs of measurements on a graph and interpret the information to determine the relationship between centimeters and inches.

L. **Making Sense of Slope**
   The handouts **B-48, 49, 50** contain five problems that are related to graphing and slopes of lines. Each problem contains a graph or chart that the students are to analyze and answer questions about. Work problem #1 as a class to give students a good idea of what is expected. It would be appropriate for students to work in groups of three or four to complete the assignment.

M. **Patterns in Area and Volume (B-39, 40, 41)**
   Students will use surface area and volume to generate data, find the algebraic expression for the pattern, and graph the data that are generated. Using blocks to build the figures is particularly helpful for many students’ understanding. There are linear, exponential, and quadratic patterns among the sequences. Only after students have described the patterns in arithmetic terms should students use the curving fitting utilities on their calculators. The method of finite differences can be used to identify quadratic patterns. (With Algebra II and Technical Math 2 students, follow up finite differences by setting up the matrix equation and solve it to derive the quadratic expression.)
N. After students have created an equation based on data, have them use it to make predictions. Here is a continuation of the example from 4.03B. What was the price of a 30-second television commercial in 2000? What was it actually? Explain any differences. Do the same for the first Super Bowl in 1967.

O. According to postal rate information, B-52, what will be the cost of a stamp in the year 2010? In what year do you predict stamps will cost 50¢?

P. **It’s All Downhill From Here** (A-23)
Create a linear model to represent rolling a ball down a ramp. Use the model to predict how far the ball will roll down a ramp of given height.

Q. Students can use the internet to find data on housing costs to graph. Have students look up data at [www.realtor.com](http://www.realtor.com) or go through their local chamber of commerce web site. They can make a table for ten houses listing the square footage of a house and the cost of a house. Using a calculator, students can draw a scatter plot of the data and find the line of best fit. Have students define and compare the slope and y-intercept for different cities or different locations in one large city. (Most often the slope would be the average cost per square foot and the y-intercept is the average price of the lot.)

R. Give students a table of data. Ask them to determine the slope and interpret. Example:

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Wages Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>28.75</td>
</tr>
<tr>
<td>10</td>
<td>57.50</td>
</tr>
<tr>
<td>15</td>
<td>86.25</td>
</tr>
<tr>
<td>20</td>
<td>115.00</td>
</tr>
</tbody>
</table>

S. **Calculator Tips** (W-35, 39, 43)

T. **Connections** (W-49)

U. **Extra Essentials** (E-25, 28, 29, 40, 48)